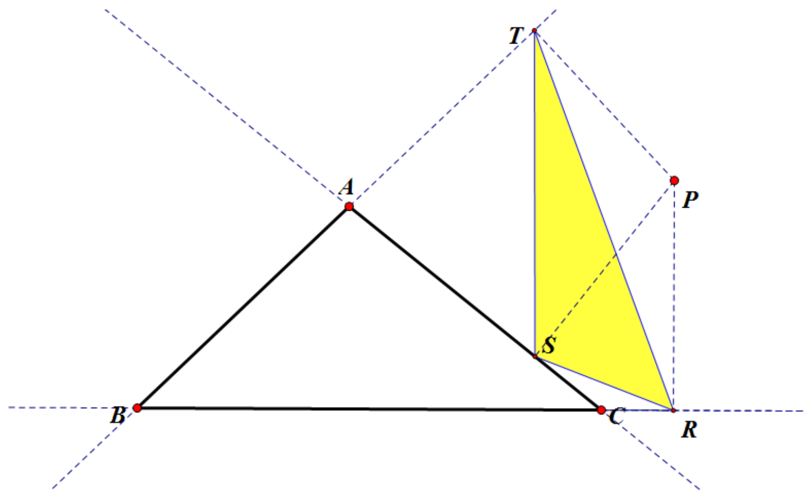
## Pedal Triangles

Definition: Let P be any point inside a given triangle ABC, and let perpendiculars PT, PR, PS be dropped to the three sides AB, BC, and CA respectively. The feet of these perpendiculars are vertices of a triangle RST which is called the *Pedal Triangle* of ΔABC for the *Interior Pedal Point* P.



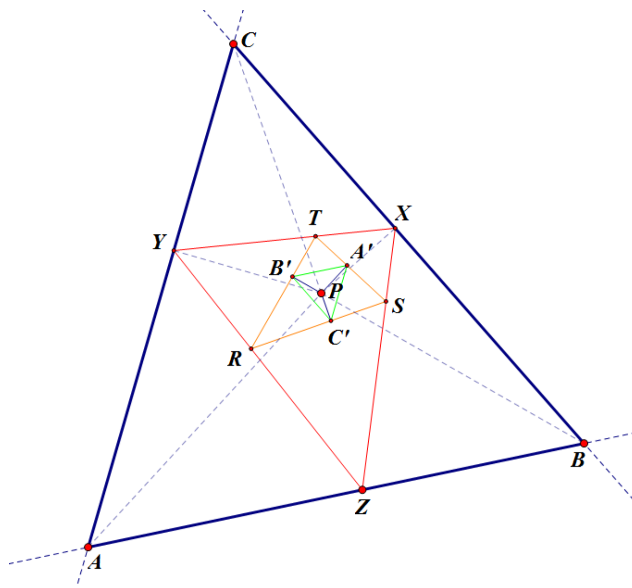

Additionally, P could be an exterior point. In that case we get the following *Pedal Triangle* of ΔABC for the *Exterior Pedal Point* P.

Note: The restriction of P to interior positions can be relaxed if we agree to insist that P will not lie on the circumcircle of ΔABC. Clearly, the orthic triangle or the medial triangle arises when P is the orthocenter or the circumcenter, respectively.

Explore the GSP file for general construction of Pedal Triangle ABC where P is ***any*** point in the plane of ABC.

Theorem: The *third pedal triangle* is similar to the original triangle.

Proof: The proof is surprisingly simple. The diagram practically gives it away, as soon as we have joined *P* to *A*.




Since *P* lies on the circumcircle of all the triangles *CXY, STX, A'C'S, RTY* and *RB'C'*, we have

$$\angle YCP = \angle YXP = \angle TXP = \angle TSP = \angle RSP = \angle A'C'P$$

and $$\angle PCX = \angle PYX = \angle PYT = \angle PRT = \angle PRB' = \angle PC'B'$$

In other words, the two parts into which *AP* divides $\angle A$, have their equal counterparts at *X* and *Y*, again at *R* and *S*, and finally both at *C'* and *B'*. Hence $\Delta ABC$ and $\Delta A'B'C'$ have equal angles at *C* and *C'*. Similarly, they have equal angles at *B* and *B'*. Thus the theorem is proved.

Note: The theorem could be further extended to prove that *the nth pedal n-gon of any n-gon is similar to the original n-gon.*